Clifford Algebras and Lie Groups workshop

Colloque "Clifford Algebras and Lie Groups for multidimensional image processing" le 2 et 3 juillet.

En savoir plus

Clifford Algebras and Lie Groups for multidimensional image processing

University of La Rochelle, 2 July - 3 July 2009

The workshop "Clifford Algebras and Lie Groups for multidimensional image processing" wild held in La Rochelle, France. The aim of this workshop is to popularize non trivial mathematical tools in the scope of multidimensional image processing.

Leading researchers will expose their works concerning the following topics:

- Mathematical aspects of Clifford Algebras
- Clifford algebras and Fourier Analysis
- Geometrical methods for diffusion : Lie groups and Clifford bundles
- Clifford extensions of the analytic signal : Hilbert transform and monogenic signal

To contact us : Christophe Saint-Jean / Renaud Péteri

This workshop is supported by:

The Office of Naval Research

The MIA laboratory
University of la Rochelle

The PRIDES federation

{mospagebreaktitle=Invited speakers}

Listof confirmed invited speakers by alphabetical order:

- Thomas Batard (University of La Rochelle, France)

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Eduardo Bayro-Corrochano (CINVESTAV, Mexico)

- Michel Berthier (University of La Rochelle, France)

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Ugo Boscain (Ecole Polytechnique, France)

- Philippe Carré (University of Poitiers, France)

- Guillaume Demarcq (University of La Rochelle, France)

- Oliver Fleischmann (Christian-Albrechts-University of Kiel, Germany)

- Laurent Fuchs (University of Poitiers, France)

- Jacques Helmstetter (Joseph Fourier-University of Grenoble, France)

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David Hestenes (Arizona State University, USA)

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Eckhard Hitzer (University of Fukui, Japan)

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Nir Sochen (University of Tel Aviv, Israel)

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Gerald Sommer (Christian-Albrechts-University of Kiel, Germany)

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Gerik Scheuermann (University of .Kaiserslautern, Germany)

{mospagebreak title=Detailed program}

Thrusday 2 July

Friday 3 July

Saturday 4 July

9h45-10:00

Conference opening

10h00-11h00

New Tools for Computational Geometry and rejuvenation of Screw Theory

David Hestenes

11H15-12h15

Clifford Algebras and multidimensional image processing

Michel Berthier

12H15-13h00

Minimal algorithms for Lipschitz monoids and Vahlen monoids

Jacques Helmstetter

14h30-15h30

Lie groups and their representations

Nir Sochen

15h45-16h45

Image reconstruction via hypoelliptic evolution on the bundle of direction of the plane

Ugo Boscain

17h00-18h00

Clifford algebras: A unifying framework for images, vector fields and orthonormal frame fields regularization

Thomas Batard

Conference Dinner

9h00-10:00

Generalized monogenic signal in conformal geometric algebra

Gerald Sommer

10h15 -11h

An Extension of the Monogenic Signal to Color Images with Applications in Image Processing

Guillaume Demarcq

11h-11h45

A Hilbert Transform on S^2 with Applications in Omnidirectional Vision

Oliver Fleischmann

12h00 -12h45

Geometric Algebras: multicomponent images analysis and geometrical modelisation

Philippe Carré and Laurent Fuchs

Lunch Break

14h00-15h00

Geometric Algebra Wavelet Transforms

Eckhard Hitzer

15h15-16h15

Convolution and Fourier transform

for vector fields

Gerik Scheuermann

16h30-17h30

How can we build an humanoid with screws?

Eduardo Bayro-Corrochano

9h30-12:30

Social event : Boat Excursion

Minimal algorithms for Lipschitz monoids andVahlen monoids

Jacques Helmstetter, Fourier Institute,Grenoble, France

An even or odd element is given in a Clifford algebra by its coordinates in some basis; does it belong to the Lipschitz monoid (that is in general the monoid or semi-group generated by all vectors)? Here an effective algorithm is given to answer this question; it is minimal in so far as then umber of equations to be satisfied is minimal. It is much faster than the classical algorithm which is only valid for an invertible element. There is an immediate application to Vahlen matrices, since the Vahlen monoid isthe image of a Lipschitz monoid.

Image reconstruction via hypoelliptic evolution on the bundle of direction of the plane

Ugo Boscain, Ecole Polytechnique, France

In this talk, I will present a method of image reconstruction based on mathematical model of human perception due to Petitot-Citti-Sarti-Agrachev.

One of the main features of this model is that the visual cortex liftsthe image from $R2$ to the bundle of direction of the plane $R2\times P1$.

Neurons are grouped into hypercolumns, each of them being a fiber of the bundle.

In this model the reconstruction is obtained by minimizing the energy necessary to activate the hypercolumns corresponding to corrupted points.

The minimization process gives rise to an hypoelliptic heat equation.

The hypoellipticity models the strong anisotropicity of the diffusion, due to the fact that groups of neurons are strongly correlated if theyare sensible to close directions in close points.

The solution of this hypoelliptic heat equation was provided using the noncommutative Fourier Transform in the paper:

A. Agrachev, U. Boscain, J.P. Gauthier, F. Rossi

“The intrinsic hypoelliptic Laplacian and the corresponding heat kernel on unimodular Lie groups”.

Journal of Functional Analysis.Volume 256, Issue 8, Pages 2621-2655.

This is a joint work with J-P Gauthier, F. Rossi and J. Duplaix.

An Extension of the Monogenic Signal to Color Images with Applications in Image Processing.

Guillaume DEMARCQ, Laboratoire Mathématiques, Image et Applications France.

In this talk, an extension of the Monogenic Signal to color images in the framework of Clifford Algebras is proposed. Using the algebra R_{5,0}, a new mathematical object is introduced called the Color Monogenic Signal. Therefore, a notion of local color phase is defined and it reveals useful in many applications. Three different applications illustrate the relevance of our approach (color segmentation, color tracking and color optical flow). Future prospects will also be discussed.

A Hilbert Transform on S^2 with Applications in Omnidirectional Vision

Oliver Fleischmann and Gerald Sommer

Cognitive Systems Group, Department of Computer Science, Kiel University, Germany

The analytic signal is an important representation in one-dimensional signal processing. Its generalization to two dimensions is the monogenic signal. The properties of the analytic and the monogenic signal in the Fourier domain are well known. A generalization to the sphere is given by the Hilbert transform on the sphere known from Clifford analysis. Nonetheless no spectral characterization exists and therefore prohibits an interpretation. We derive the spherical harmonic coefficients of the Hilbert transform on the sphere and givea series expansion. It will turn out that it acts as a differential operator on the spherical harmonic basis functions of the Laplace equation solution, analogously to the Riesz transform in two dimensions. This allows an interpretation of the Hilbert transform suitable for signal processing of signals dened on the two-sphere.The interpretation is used to derive the important features local orientation, local phase and local amplitude of intrinsically one-dimensional signals on the sphere. Additionally the Hilbert transform arises naturally from the Poisson scale space in the unit ball. This representation is justified as a novel signal model on the sphere which can be used to construct intensity and rotation invariant feature detectors in a scale-space concept.

Geometric Algebras: multicomponent images analysis and geometrical modelisation

Philippe Carré and Laurent Fuchs, XLIM-SIC, Poitiers, France.

In this presentation we will show how geometric algebras are used to develop new tools in order to analyze multicomponent images and we shall propose an algebraic theory of the global visibility problem (i.e. compute all mutually visible couples of points). First, we shall propose a study of the different Fourier transforms with the color context with the aim of an adapted filter bank definition. Then, some new developments about topological geometric modelling and discrete geometry will be discussed.

Geometric Algebra Wavelet Transforms

Eckhard Hitzer, University of Fukui, Japan.

In this presentation, it is shown how continuous Geometric Algebra Cl(n,0)-valued (n = 2,3) admissible wavelets can be constructed using the similitude group SIM(n), a subgroup of the affine group of R^n. Compared to the global GA Fourier transformations (FT) the wavelet transform allows to locally analyze vector or multivector signals. Yet the spectral GA FT representation remains an important tool for computing with GA wavelet transforms. We strictly aim for real geometric interpretation, and replace the imaginary unit i therefore with GA blades squaring to –1, specific instances of the more general geometric roots of –1. We express the admissibility condition in terms of a Cl(n,0) GA FT and then derive a set of important properties such as dilation, translation and rotation covariance, a reproducing kernel, and show how to invert the Clifford wavelet transform of vector (for n = 2) and multivector(for n = 3) functions. As concrete example, we introduce multivector GA Gabor wavelets, and describe important properties such as the GA Gabor transform isometry, a reconstruction formula, and uncertainty. We further invent a generalized GA wavelet uncertainty principle. For scalar admissibility constant, it relates frequency bandwidth and position accuracy in multivector wavelet signalandimage processing. We expect that GA wavelets will find interesting applications in signal processing and scientific visualization.

How can we build an humanoid with screws?

Prof.Eduardo Bayro-Corrochano, Departmentof Electrical Engineering and Computer Science, CINVESTAV, Guadalajara, Mexico

In thistalk we will show applications of screwtheory and Lie algebras formulated and programmed using versors in the geometricalgebra framework. This talk is a practical illustration of the first lecture of this workshop by Prof. David Hestenes titled “New Tools forComputational Geometry and rejuvenation of Screw Theory”. We focus onthe application of geometric algebra tools for the design and development of algorithms useful for perception,action, control and learning in a complex robot system like an humanoid. Inthis scenario we will show how old ideas and results of neuroscience, psychophysics,robotics, computer vision and machine learning can be nicely integrated and related in anunique computational framework such that of geometric algebra. Concepts, related equations and constrains arederived and applied to tackle challenging problems of the humanoid perception system under unexpected motion.These problems include inaccurate body-sensor calibration, difficulties toestimate ego motion and simultaneously reconstruct 3D space, image stabilization and model an human like human visionfor humanoids.

{mospagebreak title=Slides and Photos}

Presentation files :

- Thomas Batard [pdf]

- Eduardo Bayro-Corrochano [pdf] (around 300 Mb)

- Michel Berthier [pdf]

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Ugo Boscain [pdf]

- Philippe Carré [pdf]

- Guillaume Demarcq [pdf]

- Oliver Fleischman [pdf]

- Laurent Fuchs [pdf]

- Jacques Helmstetter

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David Hestenes [pdf][paper]

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Eckhard Hitzer [pdf(*)] [paper]

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Nir Sochen [pdf]

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Gerald Sommer [pdf]

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Gerik Scheuermann [pdf]

(*) : The presented work is not published yet. Ask the author for the password.

Here you can find a link to the photos of the Workshop and the Catamaran boat trip!

{mospagebreak title=Social event}

For interested people, a social event is planned on Saturday morning, which will consist in a nice boat excursion next to the islands surrounding La Rochelle. Various snacks and drinks will be served on board.

More details from our partner webpage (in french): http://www.kapalouest.com/

{mospagebreak title=Workshop location }

The workshop will take place in Amphiteater 100 at the MSI (Maison desSciences de l'ingénieur), Avenue Henri Becquerel, University of LaRochelle.

Zoom on the map

Location of the MIA laboratory:

Laboratoire de Mathénatiques, Image et Applications

Batiment Pascal, Pôle Sciences et Technologies

Université de La Rochelle

Avenue Michel Crépeau

17042 La Rochelle cedex

Tel. 33(0)5.46.45.82.01

Fax. 33(0)5.46.45.82.40

Map of La Rochelle

Map of the campus

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If you are taking the plane (La Rochelle-Ile de Ré Airport)

You can take the bus number 7 ;

and you will need tochange at Place de Verdun (bus 10, 17, 19)

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If you are coming by train

Paris (Montparnasse station) – La Rochelle : less than 3 hours;

there is also a direct train connection from Roissy Airport to la Rochelle (with a change in Poitiers)

Bordeaux – La Rochelle : 2h15

and take the bus no 10 or 17, in rue de Colmar (Colmar street) :

Unit ticket price : 1.20 €

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If you are coming by car

Take the direction « Les Minimes »: the university campus is located between the train station and the Port "Les minimes".

{mospagebreak title=Accomodation}

Here is a map of hotels located nearby the workshop or in la Rochelle city center. As July is a very touristic period in la Rochelle, we advise you to book your hotel very quickly!

Zoom on the map